Mathematics portion of the Doctor of Engineering Qualifying Examination

1. The exam will be made up by faculty members of the Department of Mathematics and Computer Science. Dr. Kathy Zhong (zhongk@udmercy.edu) is the chair of this committee.
2. The exam will be given twice a year. This will be on the Friday morning before the dead week (usually the last week of November in the Fall semester, and usually the last week of March in the Winter semester.)
3. Students in the Doctor of Engineering program will meet with their advisors to discuss the appropriate time to take the exam..
4. Student who wishes to take the Fall exam must register by September 30. Student who wishes to take the Winter exam must register by January 31.
5. **Registration procedure**: To register for the exam, send an e-mail by the deadline(s) listed above to Dr. Kathy Zhong (zhongk@udmercy.edu) in the following format:

   The **subject line** of the e-mail must contain: Engineering Qualifying Exam Winter (Fall) 20xx.

   Include the following information in the e-mail:
   Name,
   Contact information such as e-mail address and phone number
   Advisor's name
   Major
   Term (Winter or Fall) you wish to take the exam

6. The exam consists of 16 problems. Four problems are from each of the following sections:
   - **Calculus**
   - **Linear Algebra**
   - **Differential equations**
   - **Probability and statistics**

7. Students are required to do 8 problems with at least one problem from each section. Each problem will be 10 points. If more than 8 problems are selected, only the first 8 problems are graded
8. The students will have 4 hours to complete the exam. Reference books, statistics table and calculators are allowed.

Contents of the exam:

Calculus

Questions will come from material presented in both MTH 142 and 241. A doctoral student preparing for this part of the exam should read from:

Calculus: Early Transcendentals 9th Edition by Howard Anton, Irl BIvens and Stephen Davis chapters six through fifteen.

Topics to be covered:

1. Applications of Integration. Student should know the various methods for finding volumes of surfaces of revolution, arc length of curves, and surface area. See chapter 6.
2. Student should be familiar with basic notions of how to solve simple differential equations as done in Chapter 8.
3. Techniques of Integration. In particular know integration by parts, partial fraction expansion, improper integrals, trigonometric substitution and trigonometric integrals. See chapter 7.
4. Infinite Series and Sequences. Student should be familiar with all of Chapter 9. In particular finding intervals of convergence of Maclaurin and Taylor Series, derivation of Maclaurin and Taylor Series, approximating and evaluating functions via power series.
5. Students should be familiar with techniques for directional derivatives, gradient methods, maxima and minima of multivariable functions, partial derivatives, line integrals, and Greens Theorem.

Probability and Statistics:

Questions will be based on course of MTH427. The reference book is:
Probability and Statistics for Engineering and Science, by Jay Devore, latest edition.

Topics include:

- Addition and multiplication rules
- Independence of events
- Combinatorial probability
- Conditional probability
- Bayes Theorem / Law of total probability
- Univariate probability distributions (including binomial, negative binomial, geometric, hypergeometric, Poisson, uniform, exponential, chi-square, normal)
- Multivariate probability distributions (including the bivariate normal)
- Joint probability functions and joint probability density functions
- Joint cumulative distribution functions
- Central Limit Theorem

- Conditional and marginal probability distributions
- Covariance and correlation coefficients
- Descriptive statistics (mean, median, mode, percentile, variance and standard deviation)
- Statistical inferences
  - Calculate confidence intervals of population means, proportions, variances of one or two population
  - Estimate sample sizes to gain certain desired margin of error
  - Perform hypothesis testing (either classical or p-value) for means, proportions, variances of one or two population.
  - Understand the concept of type I and type II. Calculate α and β.

Differential Equations:

Questions will be based on MTH 372.
Reference Text:
An Introduction to Modern Methods and Applications, by James R. Brannan and Willam E. Boyce.

Publisher: Wiley, 2007. ISBN: 0-471-65141-9.

Topics include:

1. Linear first order equations:
The method of integrating factors. Input response models. Transient and steady-state responses. Frequency response for first order LTI systems. Principles of superposition.

2. Non-linear first order equations:
First order autonomous systems and population dynamics. Concepts of stability.

3. Numerical methods: Euler, Runge-Kutta, etc.

4. First order systems:
Reducing higher order equations to first order systems. Solving first order homogeneous linear systems with constant coefficients using the method of eigenvectors for non-defective matrices. Dynamical systems viewpoint and phase-plane analysis of trajectories for autonomous systems. Stability concepts.

5. Linear higher order equations with constant coefficients:
Characteristic polynomials, particular and general solutions, fundamental sets of solutions, and the Wronskian. The

principle of superposition. The method of undetermined
coefficients and the method of variation of parameters.

6. Second order LTI vibration theory:
The spring-mass-damper system and the series RLC circuit.
Free response and overdamped, critically damped
and underdamped systems. Quasi-frequency.
Forced sinusoidal response. LTI principles for
input response, including superposition.
Frequency response for second order LTI systems
(amplitude and phase). The Q-factor.
Resonance, for displacement and velocity
(or voltage and current). Measurement applications
and the log-decrement method.

7. Laplace transforms:
Solving ODEs using the method of Laplace transforms.
Convolution integrals. Step and impulse response.

Linear Algebra.

Questions will be based on MTH 4020.
Reference Text:
Linear Algebra and Applications, by Otto Bretscher, 3rd edition.
Publisher: Wiley, 2008. ISBN: 9788131714416

1. Vector and matrix arithmetic – vector addition, scalar multiplication, matrix multiplication.

2. Properties of vector spaces, subspaces, and linear transformations.

3. Span of a sequence of vectors, and determination of whether a sequence of vectors is linearly independent

4. Gaussian elimination – determination of whether a system of equations has zero, one, or infinitely many solutions.

5. Matrix inverses via Gauss-Jordan, and also via use of adjugate matrix

6. Determinant and trace of a matrix, eigenvalues, eigenvectors, and eigenspaces

7. Diagonalization of a matrix

Sample Exam

Doctor of Engineering Qualifying Examination
Mathematics

January, 2008

Please do 8 of the 16 problems

You must do at least one problem from each of the following 4 sections:

(a) problems 1-4
(b) problems 5-8
(c) problems 9-12
(d) problems 13-16

Show your work in the space provided

Indicate the problems you want to be graded in the following table with an X mark. If you select more than 8 problems, only the first 8 problems will be graded.

Name:______________________________

Problem	Want to be Graded	Grade	Graded by	Date
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
Total				

1. A hallway of width 6 feet meets a hallway of width 9 feet at right angles. Find the length of the longest pipe that can be carried level around this corner.

2. The Two-Point Gaussian Quadrature Approximation for *f* is

$$\int_{-1}^{1} f(x)dx \approx f(-\frac{1}{\sqrt{3}}) + f(\frac{1}{\sqrt{3}}).$$

(a) Use this formula to approximate

$$\int_{-1}^{1} \frac{1}{1+x^2}\, dx.$$

(b) Prove that Two-Point Gaussian Quadrature Approximation is exact for ALL polynomials of degrees 3 or less.

3. Let $f(x) = \sin(\ln x)$.

(a) What is the domain of the function *f*?

(b) What is the range of the function *f*?

(c) Find two values of *x* satisfying *f(x)*=-1.

(d) Find *f'(x)*.

(e) Use calculus to find the maximum value of *f* on the interval [1, 10].

4. Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$.

5. An aircraft seam requires 25 rivets. The seam will have to be reworked if any of these rivets is defective. Suppose rivets are defective independently of one another, each with the same probability.

a. If 20% of all seams need reworking, what is the probability that a rivets is defective?

b. How small should the probability of a defective be to ensure that only 10% of all seams need reworking?

6. Rockwell hardness of pins of a certain type is known to have a mean value of 50 and a standard deviation of 1.2.

a. If the distribution is normal, what is the probability that the sample mean hardness for a random sample of 9 pins is at least 51?

b. What is the (approximate) probability that the sample mean hardness for a random sample of 40 pins is at least 51?

7. On the basis of extensive tests, the yield point of a particular type of mild steel-reinforcing bar is known to be normally distributed with σ=100. The composition of the bar has been slightly modified, but the modification is not believed to have affected neither the normality nor the value of σ. Assume this to be the case, if a sample of 25 modified bars resulted in a sample average yield point of 8439 lb, compute a 90% confidence interval for the true average yield point of the modified bar.

8. The melting point of each of 16 samples of a certain brand of hydrogenated vegetable oil was determined, resulting in $\bar{x} = 94.32$. Assume that the distribution of melting point is normal with $\sigma = 1.20$.

a. Test $H_0 : \mu = 95$ versus $H_1 \neq 95$ using a two-tailed level 0.1 test.

b. If a level of 0.1 test is used, what is $\beta(94)$, the probability of type II error when $\mu = 94$?

9. Let *A* be the set of *2x2* matrices with real number entries. Let $M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Let $B = \{X \in A \mid MX = XM\}$. State precise conditions on the entries of *X* so that $X \in B$. Prove your answer. Also show that if $P, R \in B$ then both *P+R* and *PR* are elements of B.

10. Compute the eigenvalues of the of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 2 \end{pmatrix}$.

11. Let A=$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ and b=$\begin{pmatrix} 0 \\ 0 \\ 60 \end{pmatrix}$. Find the vector x so that Ax=b

12. Let λ_1, λ_2 be distinct eigenvalues of a symmetric matrix A with corresponding eigenvectors x_1, x_2. Show that both x_1 and x_2 are orthogonal to each other.

13. Solve the following system of equations. Express your answer in matrix form.

$$X' = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} X$$

14. Solve the differential equation:

$$(\sin y - y \sin x)dx + (\cos x + \cos y - y)dy = 0$$

15. Solve the initial problem

$$\frac{5}{8}\frac{d^2x}{dt^2} + 40x = 0, \quad x(0) = \frac{1}{2}, \qquad x'(0) = 0$$

16. Solve: $x^2 \dfrac{dy}{dx} + xy = 1$.