Doctor of Engineering Qualifying Examination

Mathematics

March, 2009

Please do 8 of the 16 problems

You must do at least one problem from each of the following 4 sections:

(a) problems 1-4
(b) problems 5-8
(c) problems 9-12
(d) problems 13-16

Show your work in the booklets provided

Indicate the problems you want graded

1. Let $f(x)=\frac{1}{x}$. Find the equations of the two tangent lines of $f(x)$ which pass through $(-4,3)$.

2. Use the Maclaurin Series for e^x and the Remainder theorem to determine the number of terms needed to calculate the $\sqrt{e}$ to 5 decimal places.

3. Suppose that $\sum_{k=1}^{n} u_k = 4-\frac{1}{n}$. Compute u_{100}. Does the series $\sum_{k=1}^{\infty} u_k$ converge? If it does converge, what does it converge to? Explain your reasoning.

4. Use the Method of Cylindrical shells to find the volume of the cone generated when the triangle with the vertices (0,0), (0,r), (h,0), where r>0 and h>0, is revolved about the *x*-axis.

5. A particular brand of diet margarine was analyzed to determine the level of polyunsaturated fatty acid(in percentage). A sample six packages resulted in the following data:

16.8	17.2	17.4	16.9	16.5	17.1

   a. Calculate a 99% confidence interval on the mean μ. Provide a practical interpretation of this interval.

   b. Calculate a 99% upper confidence interval on the mean. Compare this lower bound with the lower bound of the two-sided confidence interval found in step a. Discuss why they are different.

   c. State the necessary assumption about the distribution of the data to calculate the results in steps a and b.

6. A random sample of 150 recent donations at a certain blood bank reveals that 82 were type A blood. Does this suggest that the actual percentage of type A donation differs from 40%, the percentage of the population having type A blood? Carry out a test of the appropriate using a significance level of 0.1. Would your conclusion have been different if a significance level of 0.05 had been used?

7. Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Supposed a light aircraft had disappeared.
   a. If it has an emergency locator, what is the probability that it will not be discovered?

   b. If it does not have an emergency locator, what is the probability that it will be discovered?

8. The lifetime of a certain type of battery is normally distributed with mean value 10 hours and standard deviation 1 hour. There are four batteries in a package. What life time value is such that the total lifetime of all batteries in a package exceeds that value for only 5% of all packages?

9. Calculate the determinant of A, and state whether or not the matrix has an inverse:

$$A = \begin{bmatrix} 0 & -3 & 4 & 7 \\ 3 & 5 & 0 & -3 \\ 1 & 1 & 4 & 7 \\ -1 & 0 & 2 & 5 \end{bmatrix}$$

10. Given that 1 is an eigenvalue of the following matrix A, find:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}$$

a. an eigenvector corresponding to the eigenvalue 1;

b. the other two eigenvalues of A.

11. Calculate the inverse matrix (if it exists) for A:

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -4 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

12. Find all solutions (if any) to the system:

$$A = \begin{Bmatrix} 3x + 2y + 2z = 2 \\ w + 3x - y - 2z = 4 \\ -2w + 6x + 10y + 12z = 0 \\ -w + 3y + 8z = 2 \end{Bmatrix}.$$

13. Find the general solution:

$$y^{(4)} + y''' + y'' = 0$$

14. Use the Laplace Transform method to solve the Initial Value Problem:

$$y' + y = f(t), \quad y(0) = 0, \qquad \text{where } f(t) = \begin{cases} 0 & 0 \le t < 1 \\ 1 & 1 \le t < \infty \end{cases}$$

15. Consider the spring-mass-damper system in free motion satisfying

$$my'' + cy' + ky = 0$$

where m = 1, c = 2, k = 10 (with appropriate compatible units).

a. Is this system overdamped, underdamped, or critically damped?

b. If the mass is released from position y = −2 at rest, find the equation of motion.

16. Consider the Initial Value Problem

$$\frac{dy}{dx} + 2y = 2\cos 2t\,, \quad y(0) = y_0$$

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a. Solve this IVP.

b. Identify the steady state and the transient parts in the solution.